

On the Origin of Coastlines

A Study in Fractal Geometry

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Introduction

The length of a coastline depends on the length of the ruler used to measure it. As the ruler shrinks, the measured length grows without bound — an observation that first appears trivial and later turns out to reshape how we think about dimension itself.

This paper revisits the classic problem and offers a short, modern account suitable for readers encountering self-similarity for the first time.

Self-Similarity

A shape is *self-similar* when parts of it resemble the whole. Ferns, snowflakes, river networks, and — famously — coastlines all display this property across a range of scales. Zoom in on a rocky shore and each bay reveals smaller bays; each of those, smaller still. No single true length survives this scrutiny.

We can make the idea precise with a scaling exponent, the fractal dimension, which need not be a whole number. For a smooth curve it equals one; for the coast of Britain, empirical estimates place it near one and a quarter.

The Scaling Law

Lewis Fry Richardson noticed that as you shorten the measuring stick, the total measured length climbs in a strikingly regular way. Plotted on logarithmic axes, the relationship is very nearly a straight line, and the slope of that line is what we now call the fractal dimension.

Measurement in Practice

Richardson's original data came from national boundary surveys, where neighbouring countries reported strikingly different lengths for the same shared border. The discrepancy was not error but method: each survey had chosen a different ruler.

Why It Matters

The practical upshot is that length is the wrong question to ask of a rugged boundary. Dimension, not length, is the stable descriptor — a lesson that recurs throughout the natural sciences, from turbulence to the branching of blood vessels.

Conclusion

Self-similarity turns an apparent paradox — a coastline of unbounded length enclosing a finite area — into a precise, measurable statement about scale. What looks like a failure of measurement is really an invitation to measure the right thing.